Exercise 3

Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm^2?

Solution

The area of a square with side length x is

$$A = x^2.$$

Differentiate both sides with respect to t, using the chain rule on the right side.

$$\frac{d}{dt}(A) = \frac{d}{dt}(x^2)$$

$$\frac{dA}{dt} = (2x) \cdot \frac{dx}{dt}$$

The edge length is increasing by 6 centimeters per second, so $dx/dt = 6$ cm/s. Therefore, when the area of the square is 16 cm^2 (that is, when $x = 4$), the rate that area is increasing is

$$\left.\frac{dA}{dt}\right|_{x=4} = 2(4)(6) = 48 \ \frac{\text{cm}^2}{\text{s}}.$$